1. Introduction

In 1994, Los Alamos National Laboratory (LANL) pioneered the concept of microhole drilling with diameters from 1.375 in. (35 mm) to 2.375 in. (60.3 mm) for production of shallow to medium depth low-productivity reservoirs [1]. The overall goal of the microhole program is to help avoid the more complicated and labor-intensive aspects of conventional drilling and reduce the cost and pollution during drilling operations in the United States. The U.S. Department of Energy estimates that two thirds of all oil discovered in the U.S. remains in the ground. Of this oil, half (218 billion barrels) is in reservoirs shallower than 5000 feet [2]. The lower cost of microhole coiled tubing drilling technology can make these untapped reserves economically viable.

Microhole drilling technology has many advantages, including greatly reduced tripping time, significantly improved drilling efficiency, saved drilling time and cost, simplified pressure control during drilling, and lower labor and material requirements, when compared with conventional drilling [3-5]. However, the biggest shortcomings associated with microhole drilling technology are, that, the annulus of microhole drilling is smaller and that there is no rotation of drilling stem. Both of these factors significantly reduce the hole cleaning and sharply increase the friction between the drilling stem and the bore wall, thus hindering microhole-drilling efficiency. More seriously, cuttings accumulation may cause sticking which will lead to microhole drilling accidents. Therefore, it is necessary to study the critical fluid velocity required for hole cleaning to effectively reduce friction in inclined and horizontal sections during microhole drilling and improve drilling efficiency and quality.

Considerable work has been done to study the nonstationary motion of two-phase fluid in pipes. The first work on transient models for cuttings removal was presented by Iyoho et al. in 1988 [6]. In 1999, Martins et al. [7] (SPE 56560) presented a transient model for horizontal wells using a two-layer model and determined formulas from the mass and momentum conservation equations. In 2006, Shigemi Naganawa [8] developed a modified two-layer model to simulate transient behavior of cuttings transport over the entire trajectory of an extended-reach well. In 2008, Suzana Santos Costa et al. [9] presented a new time-dependent model to simulate cuttings transportation and ECD in wellbore drilling. All these works focused on developing predictions for cuttings concentration, bed height, and other parameters in nonstationary flow. However, some researchers, such as Ford, Ramadan and Longqing Zou [10-12], focused on the research of critical velocity for hole cleaning. Based on their work, a mathematical model of critical velocity for cuttings transport in microhole drilling was proposed in this article.

2. Cuttings Transport Mechanisms

In vertical and nearly vertical sections of micro wellbore (inclination angles less than 10°), cuttings tend to sink axially along the wellbore due to gravity effects. However, the flow direction of annulus drilling
mud is contrary to that of cuttings. The cuttings will experience a drag force and finally come to a constant velocity which is defined as the terminal settling velocity in drilling mud. As the settling direction of cuttings is contrary to that of drilling mud velocity in the same line, cuttings will be slowly carried out of the micro wellbore by mud as long as the upward velocity of mud is slightly greater than the terminal settling velocity of cuttings. This process ensures the continuous circulation of mud during microhole drilling.

In inclined and horizontal sections of micro wellbores, the direction of particle gravity and mud flow direction are not in the same line. Cuttings tend to deposit along the axial direction to the bottom of micro wellbore under the effect of the axial component of the gravitational force. However, cuttings tend to deposit on the low side of the wellbore under the effect of the radial component of the gravitational force. When the mud displacement is high enough, cuttings in a micro wellbore will move into suspension under the influence of lift force and radial pulsations of mud. With the reduction of mud flow rate, the fluid forces acting on the cuttings are not enough to overcome the gravitational force and wellbore friction. If this occurs, some cuttings may be transported through tumbling and saltation on the lower side of the micro wellbore and form a moving bed. However, as the fluid flow rate is further reduced, the thickness of the sedimentary bed increases and the fluid drag force cannot act on the bottom of the cuttings bed. Then, a stationary cuttings bed will form on the lower side of the micro wellbore and newly generated cuttings will be mainly transported through a flow region upon the bed by rolling and suspension. The cuttings bed in the inclined and horizontal segments of micro wellbore will increase the pressed area of the drilling stem, which will lead to an increase of friction, thereby affecting the drilling efficiency of micro wellbore. Sometimes, sticking accidents may occur if the bed becomes too thick and buries the drilling stem.

3. Basic assumptions

Mathematical modeling of solid particle transport phenomena in pipes is quite complicated and requires a considerable idealization of not only the hydrodynamics in the fluid flow, but also the characteristics of the cuttings. By representing non-uniform cuttings, bed properties, and flow properties by their mean values, we can reduce the difficulty of defining the critical velocity variation along the wellbore.

1. The fluid is in fully developed flow without fluctuation. The cuttings do not influence the fluid velocity and viscosity.
2. The cuttings can be represented by a spherical particle with uniform size, density, and angle of friction.
3. The sedimentary bed on the lower side of pipe has a uniform thickness and uniform arrangement.
4. Particle force analysis is simplified by neglecting the micro forces such as plastic force, pressure force, added mass force, and Basset force.

4. Model Building

It is assumed for this model that there is a stationary sedimentary bed with a uniform arrangement on the low side of pipes (fig.1), and that the repose angle of cuttings $\Phi$ is assumed to be $30^\circ$. The interaction between cuttings and fluid is realized through momentum exchange, and the key steps to analyzing the motion state of single particle are determining the forces acting on the particle and studying the force and moment balances on it. When the hydrodynamic forces exerted by fluid flowing over a particle on sedimentary bed surface exceed its static forces, the particle begins moving. Therefore, the forces acting on a particle should be studied before determining its motion state.

Both gravitational force and buoyancy force act on the solid particle which sediments in fluid. The difference between these two forces is referred to as net weight.

$$F_G = \pi d_p^2 (\rho_s - \rho_f) g / 6 \tag{1}$$

In eq.1, $d_p$ is the diameter of cuttings, $\rho_s$ is the density of cuttings, $\rho_f$ is the density of the fluid.

Based on the study by Landau and Lifshitz [13], when there is relative movement between the particle and its surrounding fluid, the particle will experience a drag force induced by pressure and shear stress acting on the particle surface. Since drilling cuttings are relatively small, it is appropriate to consider the condition of a small Reynolds number. It is well known that flow rate is proportional to the square of the velocity in the convection of momentum, and momentum transport in laminar flow is the linear function of velocity. Therefore, in the case of a small Reynolds number, the flow rate of convection momentum can be ignored. If the Reynolds number is less than 1, the term $\nu \Delta v$ in the Navier-Stokes equation can be omitted. In the case of steady flow, the local acceleration term will disappear. With ignorance of body force, the Navier-Stokes equation becomes:

$$\frac{1}{\rho_f} \nabla p = \nu \Delta v \tag{2}$$

![Fig.1. Cuttings arrangement and the main forces acting on a surface particle in a micro wellbore](image-url)
When considering the geometry of the cuttings, it is more convenient to use spherical coordinates to express equation. If the velocity is assumed to be in the radial and circumferential directions only and has nothing to do with azimuth coordinates which means \( v_r = v_\theta = v_\phi = 0 \), then

\[
\frac{\partial v_r}{\partial \phi} = \frac{\partial v_r}{\partial \phi} = 0
\]  

(3)

In spherical coordinates, the simplified expression of Navier-Stokes equation is

\[
\frac{1}{\rho_r} \frac{\partial p}{\partial r} + \frac{\partial v_r}{\partial r} = \frac{v_r^2}{\rho_r} + \frac{v_\theta^2}{\rho_r} + \frac{v_\phi^2}{\rho_r} + \frac{1}{\rho_r} \frac{\partial}{\partial \theta} \left( \frac{\rho_r v_r v_\theta}{\rho_r} \right) + \frac{1}{\rho_r} \frac{\partial}{\partial \phi} \left( \frac{\rho_r v_r v_\phi}{\rho_r} \right)
\]

(4)

\[
\frac{1}{\rho_r} \frac{\partial p}{\partial r} = \frac{v_r^2}{\rho_r} + \frac{v_\theta^2}{\rho_r} + \frac{v_\phi^2}{\rho_r} + \frac{1}{\rho_r} \frac{\partial}{\partial \theta} \left( \frac{\rho_r v_r v_\theta}{\rho_r} \right) + \frac{1}{\rho_r} \frac{\partial}{\partial \phi} \left( \frac{\rho_r v_r v_\phi}{\rho_r} \right)
\]

(5)

\[
\frac{\partial v_r}{\partial r} + \frac{2v_r}{r} + \frac{v_\theta}{r \cos \theta} = 0
\]

(6)

In these equations, \( \cot \theta = \cos \theta / \sin \theta \), with boundary conditions: (1) \( \theta = 0 \); (2) \( \theta, (\theta = 0, \phi = 0) = u_c \), \( \cos \theta \); \( v_r (\theta, \phi = 0) = -u_c \sin \theta \), which means velocity components in the cuttings surface \( v_r \) & \( v_\theta \) are 0, and the velocity \( v_r \) is equal to a constant \( u_c \) at infinity. \( u_c \) is the relative velocity between fluid and cuttings.

After consolidation and differentiation, we will get the component velocity and pressure:

\[
v_r = \left[ 1 - \frac{3}{2} \frac{R}{r} + \frac{1}{2} \frac{R^2}{r^2} \right] u_c \cos \theta
\]

(7)

\[
v_\theta = \left[ 1 - \frac{3}{4} \frac{R}{r} + \frac{1}{4} \frac{R^2}{r^2} \right] u_c \sin \theta
\]

(8)

\[
p - p_c = -\frac{3}{2} \rho v \frac{R}{r} \cos \theta
\]

(9)

The force acting on the surface of the cuttings comes from dynamic pressure and shear stress. The component in the direction of velocity \( u_c \) is called drag force:

\[
F = \int \rho v \cos \theta dA + \tau_s \sin \theta dA
\]

(10)

\[
= \frac{3}{2} \rho v \frac{R}{K} \cos \theta 2 \pi \sin \theta d \theta + + 3 \pi \rho v u \sin' \theta d \theta = 6 \pi \rho v u
\]

In eq.10, \( R \) is the radius of cuttings, \( \rho \) is the density of fluid, \( v \) is the kinematic viscosity, and \( u_c \) is the relative velocity between cuttings and fluid.

Although all the parameters are known in eq.10, as shown in figure 1, it is only applicable when the Reynolds number is less than 1.

Substituting drag coefficient \( C_D = \frac{24}{Re} \) and Reynolds number \( Re = \frac{\rho u d}{\mu} \), eq.10 becomes a general formula:

\[
F = 6\pi \rho \frac{u d}{d} \frac{C_D}{24} \times u_c = \frac{1}{8} \pi d^4 \rho u_c
\]

(11)

In eq.11, \( d_c = 2R \) is the diameter of cuttings and \( \mu = \rho v \) is the dynamic viscosity of the fluid. From eq.11 we can see that the key step to calculating drag force is to determine the drag coefficient \( C_D \). Voloshuk [14] and Phillip [15] determined the drag coefficient values of single particle with different particle Reynolds numbers via experimental study(fig.4). Based on these experimental data, we found that the formula proposed by Turton [16] acquired the minimum relative error (only 3.06% when compared with experimental data). Therefore, Turton’s formula is applied for this drag coefficient calculation.

\[
C_D = 24 \left( 1 + 0.173 \frac{Re}{Re} + 0.413 \right) \left( 1 + 16300 \frac{Re}{Re} \right)
\]

(12)

Fluid shear can exert a lift force, known as Saffman force, on the particle in solid-liquid two-phase flow. Saffman [17] first found that a particle would experience the lift force during transportation in viscous fluid. Based on the assumptions of unbounded linear shear flow and constant velocity gradient, he derived a formula to estimate the lift force via the Navier-Stokes equation:

\[
F_l = 6.46 \rho u d^4 (\frac{dv}{dy})^2
\]

(13)

In equation (13), \( v \) is kinematic viscosity of fluid and \( u_c \) is relative fluid-particle velocity.

However, the correlation is only valid for particles transportation in no-boundary fluid flow and cannot be used for particles on or near a surface. On the analogy of the drag force calculation method, Clark and Bickman [18] proposed that the lift force correlation could be calculated with

\[
F_l = C_l \pi d^4 \rho u_c^4 / 8
\]

(14)

Furthermore, they introduced a lift coefficient for the particle resting on the sedimentary bed. The lift coefficient is given as follows:

\[
C_l = 0.178
\]

(15)

or

\[
C_l = 5.82(a_0 / Re)^{1/2}
\]

(16)

In eq.16, \( a_0 \) is the inclination angle of wellbore and \( \Phi \) is the repose angle of the cuttings as shown in figure 1.

Substituting eq.1, 11 and 14 into eq.8, we will obtain the minimum rolling transportation velocity:
In eq.(18), $s = \rho_f / \rho_p$.
When the fluid flow rate increases to a certain value, particles on bed surface will be suspended and move into suspension transportation. As shown in Figure 1, assuming a surface particle is on the threshold of suspension, the balance of forces in the X-Y directions can be separately expressed as:

$$F_x = F_c \cos a$$
$$F_y = F_c \sin a$$

Substituting eq.1, 11, and 14 into eq.19 and 20, we will obtain the minimum suspension transportation velocity in y-x directions respectively:

$$u_{sw} = \left[ \frac{4d_p g (s - 1) \cos \alpha}{(3C_d)} \right]^{1/2}$$
$$u_{sw} = \left[ \frac{4d_p g (s - 1) \sin \alpha}{(3C_d)} \right]^{1/2}$$

Analyzing eq.21 and 22 we can obtain:

$$u_{sw} = \frac{\tan \alpha C_d}{C_i}$$
$$u_{sw} = \max [u_{sw}, u_{uw}]$$

Therefore, based on cuttings transport mechanisms, the relative velocity for cuttings transport is

$$v_i = \min [u_{sw}, u_{uw}]$$

Augusto et al. [19] found that the average cuttings lag was nearly 40% of the fluid velocity in horizontal and deviated wells. Therefore, the critical fluid velocity to guarantee the results of a sand washing project in a horizontal well should be:

$$v_i = v_i / 40\%$$

In eq.15, $v_i$ is the critical fluid velocity for cuttings transport (m/s) and $v_i$ is the relative velocity between cuttings and fluid calculated by the model (m/s).

5. Results and analysis

Four rigid spherical particles having diameters 0.5, 1, 3, 5 mm, and a density of 2700 kg/m$^3$ have been used in numerical calculation. The three types of fluids used were water, drilling mud 1, and drilling mud 2. These fluids had densities of 1000 kg/m$^3$, 1150 kg/m$^3$, and 1150 kg/m$^3$, respectively. The fluid viscosities were 0.001 Pa·s, 0.01 Pa·s, and 0.03 Pa·s, respectively at 20$^\circ$C. Table 1 lists all physical properties of particles and fluids used in mathematical calculation.

For cuttings suspension transport, according to the proposed mechanistic model and eq.21-22, the critical fluid velocity in the directions of X-Y axis (axial and radial directions of micro wellbore) when the inclination angle ranges from 0$^\circ$ to 90$^\circ$, is shown in figure 2. With the increase of the inclination angle, the minimum suspension velocity in the X-axis direction decreases, while the Y-axis direction increases. Furthermore, there is an intersection of the curves of critical suspension velocity in the X-Y axis when the inclination angle is relatively small. Based on eq.23, the location for the intersection is decided by the drag and lift coefficient and the inclination angle. Only the plot above the intersection keeps the cuttings in the suspension status.

For cuttings rolling transport, by analyzing eq.18, we obtain the critical fluid velocity for cuttings in the state of rolling migration with a different inclination angle. As shown in figure 3, we obtain the relationship between the inclination angle and the critical velocity for suspension or rolling, combined with critical fluid velocity for cuttings suspension transport. It is clear

<table>
<thead>
<tr>
<th>Physical properties of test particles, fluids and pipes</th>
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<tbody>
<tr>
<td><strong>Particle diameter, mm</strong></td>
</tr>
<tr>
<td><strong>Particle density, kg/m$^3$</strong></td>
</tr>
<tr>
<td><strong>Fluid density, kg/m$^3$</strong></td>
</tr>
<tr>
<td><strong>Drilling mud 1</strong> 1150</td>
</tr>
<tr>
<td><strong>Drilling mud 2</strong> 0.01</td>
</tr>
<tr>
<td><strong>Drilling mud 2</strong> 0.03</td>
</tr>
</tbody>
</table>

![Fig.2. Pattern of the critical suspension velocity of cuttings in the X-Y axis in a micro wellbore](image)

![Fig.3. Pattern of the critical velocity of the rolling and suspension transport of cuttings in a micro wellbore](image)
that the suspension transport mechanism dominates particle transportation at low inclination angles (usually less than 30°), while the rolling transport mechanism dominates particle transportation in the range of medium angles to a horizontal condition. These findings coincide with the particle transport mechanisms analyzed earlier in this paper. However, it should be noted that it is merely theoretical to assume that a single mechanism dominates cuttings transportation in a certain time frame. Because of the complexity of the flow, the two mechanisms may appear practically simultaneously.

Figures 4, 5, 6, and 7 show critical velocity for cuttings with diameters of 0.5 mm, 1 mm, 3 mm, and 5 mm in different drilling fluids with different viscosities during microhole drilling. For cuttings with a certain diameter, there is an apparent decrease in the critical velocity for microhole cuttings cleaning with the increase of the viscosity of the fluid. This phenomenon is due to the fact that the fluid forces acting on the cuttings increase significantly with viscosity, which allows the cuttings to overcome gravity and move. We can also observe that with a certain cuttings size, the critical velocity for suspension at a low inclination angle drops more sharply than the critical velocity for rolling at a medium inclination angle with the increase of viscosity. This is because the greater viscosity is more conducive to suspending cuttings when the critical suspension velocity is greatly reduced.

Figures 8, 9, and 10 show the critical fluid velocity for transport of cuttings with four different diameters during microhole drilling, using water, drilling mud 1, and drilling mud 2, respectively.
of the cuttings diameter, there is a distinct increase in the critical fluid velocity for the cleaning of micro wellbore cuttings. When the inclination angle increases, the critical velocity decreases, then increases, reaching the maximum angle around 60°. In microhole drilling, it is difficult for cuttings to be transported when the inclination angle ranges from 50-70°, as the drag and lift forces reach their minimum values almost simultaneously. Also, the critical fluid velocity for transporting cuttings reaches its maximum value in this range. As a result, the cuttings bed develops, causing friction between the drilling stem and the bore wall, both of which lead to a significant decrease in drilling efficiency. As shown in the figures below, critical velocity for cuttings transport reaches a minimum velocity when the inclination angle is within 0-30°. According to eq.21, 22 and 23, the exact location depends on the drag coefficient, lift coefficient and inclination angle.

6. Field Verification

The drilling data of a microhole horizontal well at China’s Daqing oilfield when drilling to 3400 m are as follows: drilling fluid displacement 0.172 m³/min, drill bit diameter 88.9 mm, coiled tubing outer diameter 60.3 mm, drilling mud density 1250 kg/m³, drilling mud viscosity 28 mPa*s, cuttings maximum diameter 5.8 mm, cuttings density 2300 kg/m³.

The mathematical model of critical velocity proposed in this article is applied to analyze critical velocity for cuttings transport in the microhole horizontal well, while the required drilling fluid displacement for cuttings transport at different depths is shown in figure 11. The prediction of the mathematical model indicates that the critical displacement is less than the actual operating displacement. Therefore, it can be ascertained that there will be no cuttings accumulation during the microhole horizontal well drilling with drilling fluid displacement 0.172 m³/min. This conclusion agrees with the observation that there was no unusual phenomenon of hole cleaning during the drilling process. It also indicates that the model proposed in this article can be used for analysis and guidance of microhole drilling.
7. Conclusions

There are two main mechanisms for cuttings transport during microhole drilling: the suspension transport mechanism and the rolling transport mechanism. The suspension transport mechanism takes precedence at low inclination angles; in the range of medium inclination angles to a horizontal condition, the rolling transport mechanism dominates cuttings transport when fluid velocity is low. Meanwhile, the suspension transport mechanism gradually comes to domination as the fluid velocity increases. A mathematical model of critical fluid velocity for cuttings transport in micro wellbore was proposed in this paper. Model predictions indicate that the critical fluid velocity reaches a minimum value at a low inclination angle. The position of minimum value is determined by the drag coefficient, the lift coefficient, and the inclination angle. The critical fluid velocity for cuttings transport reaches its peak value within the scope of 50-70°inclination angles during microhole drilling. This increase in velocity will sharply increase the friction between the drilling stem and the bore wall, causing the microhole drilling efficiency to be affected. According to field operation and model predictions, the increase of drilling fluid viscosity tends to induce the critical fluid velocity for cuttings transport in the micro wellbore. However, as the pressure loss in the micro wellbore is large, the pressure in the well should also be considered to select a reasonable viscosity.

References


Математическая модель критической скорости выноса бурового шлама при бурении скважин малого диаметра

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Реферат

В статье анализируются основные механизмы выноса бурового шлама в процессе бурения скважин малого диаметра и предлагается математическая модель критической скорости жидкости для выноса бурового шлама. Показано, что минимальное значение критическая скорость жидкости достигает при малых углах наклона, а пикового значения в пределах 50-70 градусов от угла наклона скважины. Увеличение вязкости бурового раствора, как правило, приводит к увеличению критической скорости жидкости для выноса бурового шлама. Однако, так как потери давления в микролюбле могут быть большими, это также необходимо учитывать при выборе приемлемой вязкости жидкости.

Kiçik diameetrlı quyuların qazıması zamanı qazma şlamanın çixarılmasının kritik sürətinin riyazi modeli

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(Çın Neft Universiteti)

Xülasə

Maqalədə quyuların qazıması prosesində qazma şlamanın çixarılmasının asas mehanizmları təhlil edilir və qazma şlamlarının çixarılmasının üçün mayenin kritik sürətinin riyazi modeli təklik olunur. Göstərilir ki, mayenin kritik sürət kicik bucaq altında aylıma zamanı minimal qiymətə çatır, quyunun aylıma bucaqından 50-70 dərəcə haddində isə pik qiymətə çatır. Qazma mənələrinin özluəyyünün artırması, bir qayda olaraq qazma şlamanın çixarılmasının üçün mayenin kritik sürətinin artırmasına gətirib çxarır. Lakin, mikrolülyada toziyiq itkisi böyük ola bilar ki, bunu münəsib maye özluəyyünün seçimində nəzərə almaq vacibdir.